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Numerical Analysis (10th Edition)

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Problem

Repeat Exercise 5 with $TOL = 10^{-3}$ in the l_∞ norm. Compare the results in parts (b) and (c) to those obtained in Exercises 5 and 7 of Section 7.3 and Exercise 5 of Section 7.4.

Reference: Exercise 5

Perform only two steps of the conjugate gradient method with $C = C^{-1} = I$ on each of the following linear systems. Compare the results in parts (b) and (c) to the results obtained in parts (b) and (c) of Exercise 1 of Section 7.3 and Exercise 1 of Section 7.4.

a.
$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1,\\ -x_1 + 6x_2 + 2x_3 &= 0,\\ x_1 + 2x_2 + 7x_3 &= 4. \end{aligned}$$

b.
$$\begin{aligned} 10x_1 - x_2 &= 9,\\ -x_1 + 10x_2 - 2x_3 &= 7,\\ -2x_2 + 10x_3 &= 6. \end{aligned}$$

Reference: Exercise 1 of Section 7.3

Find the first two iterations of the Jacobi method for the following linear systems, using $\mathbf{x}^{(0)} = \mathbf{0}$:

a.
$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1,\\ 3x_1 + 6x_2 + 2x_3 &= 0,\\ 3x_1 + 3x_2 + 7x_3 &= 4. \end{aligned}$$

b.
$$\begin{aligned} &10x_1 - x_2\\ &-x_1 + 10x_2 - 2\\ &-2x_2 + 10 \end{aligned}$$

c.
$$\begin{aligned} 10x_1 + 5x_2 &= 6,\\ 5x_1 + 10x_2 - 4x_3 &= 25,\\ -4x_2 + 8x_3 - x_4 &= -11,\\ -x_3 + 5x_4 &= -11. \end{aligned}$$

d.
$$\begin{aligned} &4x_1 + x_2 + x_3\\ &-x_1 - 3x_2 + x_3\\ &2x_1 + x_2 + 5x_3\\ &-x_1 - x_2 - x_3\\ &2x_2 - x_3 \end{aligned}$$

Reference: Exercise 1 of Section 7.4

Find the first two iterations of the SOR method with $\omega = 1.1$ for the following linear systems, using

$\mathbf{x}^{(0)} = \mathbf{0}$:

a.
$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1,\\ 3x_1 + 6x_2 + 2x_3 &= 0,\\ 3x_1 + 3x_2 + 7x_3 &= 4. \end{aligned}$$

b.
$$\begin{aligned} 10x_1 - x_2 &= 9,\\ -x_1 + 10x_2 - 2x_3 &= 7,\\ -2x_2 + 10x_3 &= 6. \end{aligned}$$

c.
$$\begin{aligned} 10x_1 + 5x_2 &= 6,\\ 5x_1 + 10x_2 - 4x_3 &= 25,\\ -4x_2 + 8x_3 - x_4 &= -11,\\ -x_3 + 5x_4 &= -11. \end{aligned}$$

d.
$$\begin{aligned} 4x_1 + x_2 + x_3 + x_5 &=\\ -x_1 - 3x_2 + x_3 + x_4 &=\\ 2x_1 + x_2 + 5x_3 - x_4 - x_5 &=\\ -x_1 - x_2 - x_3 + 4x_4 &=\\ 2x_2 - x_3 + x_4 + 4x_5 &= \end{aligned}$$

Reference: Exercise 5 of Section 7.3

Use the Jacobi method to solve the linear systems in Exercise 1, with $TOL = 10^{-3}$ in the l_∞ norm.

Reference: Exercise 1

Find the first two iterations of the Jacobi method for the following linear systems, using $\mathbf{x}^{(0)} = \mathbf{0}$:

a.
$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1,\\ 3x_1 + 6x_2 + 2x_3 &= 0,\\ 3x_1 + 3x_2 + 7x_3 &= 4. \end{aligned}$$

b.
$$\begin{aligned} &10x_1 - x_2\\ &-x_1 + 10x_2 - 2\\ &-2x_2 + 10 \end{aligned}$$

c.
$$\begin{aligned} 10x_1 + 5x_2 &= 6,\\ 5x_1 + 10x_2 - 4x_3 &= 25,\\ -4x_2 + 8x_3 - x_4 &= -11,\\ -x_3 + 5x_4 &= -11. \end{aligned}$$

d.
$$\begin{aligned} &4x_1 + x_2 + x_3\\ &-x_1 - 3x_2 + x_3\\ &2x_1 + x_2 + 5x_3\\ &-x_1 - x_2 - x_3\\ &2x_2 - x_3 \end{aligned}$$

Reference: Exercise 7 of Section 7.3

Use the Gauss-Seidel method to solve the linear systems in Exercise 1, with $TOL = 10^{-3}$ in the l_∞ norm.

Reference: Exercise 1

Find the first two iterations of the Jacobi method for the following linear systems, using $\mathbf{x}^{(0)} = \mathbf{0}$:

a.
$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1,\\ 3x_1 + 6x_2 + 2x_3 &= 0,\\ 3x_1 + 3x_2 + 7x_3 &= 4. \end{aligned}$$

b.
$$\begin{aligned} &10x_1 - x_2\\ &-x_1 + 10x_2 - 2\\ &-2x_2 + 10 \end{aligned}$$

c.
$$\begin{aligned} 10x_1 + 5x_2 &= 6,\\ 5x_1 + 10x_2 - 4x_3 &= 25,\\ -4x_2 + 8x_3 - x_4 &= -11,\\ -x_3 + 5x_4 &= -11. \end{aligned}$$

d.
$$\begin{aligned} &4x_1 + x_2 + x_3\\ &-x_1 - 3x_2 + x_3\\ &2x_1 + x_2 + 5x_3\\ &-x_1 - x_2 - x_3\\ &2x_2 - x_3 \end{aligned}$$

Reference: Exercise 5 of Section 7.4

Use the SOR method with $\omega = 1.2$ to solve the linear systems in Exercise 1 with a tolerance $TOL = 10^{-3}$ in the l_∞ norm.

Reference: Exercise 1

Find the first two iterations of the SOR method with $\omega = 1.1$ for the following linear systems, using

$\mathbf{x}^{(0)} = \mathbf{0}$:

a.
$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1,\\ 3x_1 + 6x_2 + 2x_3 &= 0,\\ 3x_1 + 3x_2 + 7x_3 &= 4. \end{aligned}$$

b.
$$\begin{aligned} 10x_1 - x_2 &= 9,\\ -x_1 + 10x_2 - 2x_3 &= 7,\\ -2x_2 + 10x_3 &= 6. \end{aligned}$$

c.
$$\begin{aligned} 10x_1 + 5x_2 &= 6,\\ 5x_1 + 10x_2 - 4x_3 &= 25,\\ -4x_2 + 8x_3 - x_4 &= -11,\\ -x_3 + 5x_4 &= -11. \end{aligned}$$

d.
$$\begin{aligned} 4x_1 + x_2 + x_3 + x_5 &=\\ -x_1 - 3x_2 + x_3 + x_4 &=\\ 2x_1 + x_2 + 5x_3 - x_4 - x_5 &=\\ -x_1 - x_2 - x_3 + 4x_4 &=\\ 2x_2 - x_3 + x_4 + 4x_5 &= \end{aligned}$$

Step-by-step solution

Step 1 of 43

Claim: To solve the given linear system using conjugate gradient method $(C = C^{-1} = I)$ **with ten-digit rounding arithmetic.**

The **conjugate gradient method** of Hestenes and Stiefel is a direst method designed to solve a $n \times n$ positive definite linear system. It is very useful when employed as an iterative approximation method for solving large sparse systems with nonzero entries. When the matrix is preconditioned to make the calculations more effective, good results are obtained in only about $\sqrt{n}$ iterations.

Comment

Step 2 of 43

Consider $\mathbf{x}^{(0)} = (0,0,0)^t$ as an initial approximation to the exact solution.

The conjugate gradient method involves the computation of the following Equations.

$$\left\{\begin{array}{l} \mathbf{r}^{(0)} = \mathbf{b} - A\mathbf{x}^{(0)}; \\ \mathbf{v}^{(1)} = \mathbf{r}^{(0)} \end{array}\right\} \text{ and for } k = 1,2,3,\cdots,n \left\{\begin{array}{c} t_k = \dfrac{\langle \mathbf{r}^{(k-1)}, \mathbf{r}^{(k-1)} \rangle}{\langle \mathbf{v}^{(k)}, A\mathbf{v}^{(k)} \rangle}, \\ \mathbf{x}^{(k)} = \mathbf{x}^{(k-1)} + t_k \mathbf{v}^{(k)}, \\ \mathbf{r}^{(k)} = \mathbf{r}^{(k-1)} - t_k A\mathbf{v}^{(k)}, \\ s_k = \dfrac{\langle \mathbf{r}^{(k)}, \mathbf{r}^{(k)} \rangle}{\langle \mathbf{r}^{(k-1)}, \mathbf{r}^{(k-1)} \rangle}, \text{ and} \\ \mathbf{v}^{(k+1)} = \mathbf{r}^{(k-1)} + s_k \mathbf{v}^{(k)} \end{array}\right\}$$

Comment

Step 3 of 43

a. Consider the linear system of equations $\left\{\begin{array}{c} 3x_1 - x_2 + x_3 = 1, \\ -x_1 + 6x_2 + 2x_3 = 0, \\ x_1 + 2x_2 + 7x_3 = 4 \end{array}\right\}$

Use Maple to solve the linear system with $TOL = 10^{-3}$ in the l_∞ norm

Step1: compute the first iteration when $k = 1$

$> Tolerance := 10^{-3}:$

$> with(LinearAlgebra):$

$> A := Matrix([[3,-1,1],[-1,6,2],[1,2,7]]):$

$> b := \langle 1,0,4 \rangle:$

$> C := Matrix([[1,0,0],[0,1,0],[0,0,1]]):$

$> B := Transpose(C):$

Comment

Step 4 of 43

Continuation of the above

$> x_0 := \langle 0,0,0 \rangle:$

$> r_0 := b - A.x_0:$

$> w_1 := C.r_0:$

$> v_1 := B.w_1:$

$> \alpha_1 := DotProduct(w_1, w_1):$

$> u_1 := A.v_1:$

$> t_1 := \dfrac{\alpha_1}{DotProduct(v_1, u_1)}:$

$> x_1 := x_0 + t_1 \cdot v_1:$

$> x1 := evalf(x_1)$

$$x1 := \begin{bmatrix} 0.1382113821 \\ 0. \\ 0.5528455285 \end{bmatrix}$$

Comment

Step 5 of 43

Continuation of the above

$> r_1 := r_0 - t_1 \cdot u_1:$

$> r1 := evalf(Norm(r_1, infinity))$

$r1 := 0.9674796748$

$> w_2 := C.r_1:$

$> \alpha_2 := DotProduct(w_2, w_2):$

$> s_1 := \dfrac{\alpha_2}{\alpha_1}:$

$> v_2 := B.w_2 + s_1 \cdot v_1:$

$> v2 := evalf(v_2)$

$$v2 := \begin{bmatrix} 0.08764624232 \\ -0.9674796748 \\ 0.2123735872 \end{bmatrix}$$

Step2: compute the second iteration when $k = 2$

$> u_2 := A.v_2:$

$> t_2 := \dfrac{\alpha_2}{DotProduct(v_2, u_2)}:$

$> x_2 := x_1 + t_2 \cdot v_2:$

$> x2 := evalf(x_2)$

$$x2 := \begin{bmatrix} 0.1535933455 \\ -0.1697932118 \\ 0.5901172091 \end{bmatrix}$$

Continuation of the above

$> r_2 := r_1 - t_2 \cdot u_2:$

$> r2 := evalf(Norm(r_2, infinity))$

$r2 := 0.2206904575$

$> w_3 := C.r_2:$

$> \alpha_3 := DotProduct(w_3, w_3):$

$> s_2 := \dfrac{\alpha_3}{\alpha_2}:$

$> v_3 := B.w_3 + s_2 \cdot v_2:$

$> v3 := evalf(v_3)$

$$v3 := \begin{bmatrix} -0.2158448788 \\ -0.06136953596 \\ 0.06691382426 \end{bmatrix}$$

Comment

Step 6 of 43

Step3: compute the third iteration when $k = 3$

$> u_3 := A.v_3:$

$> t_3 := \dfrac{\alpha_3}{DotProduct(v_3, u_3)}:$

$> x_3 := x_2 + t_3 \cdot v_3:$

$> x3 := evalf(x_3)$

$$x3 := \begin{bmatrix} 0.06185567010 \\ -0.1958762887 \\ 0.6185567010 \end{bmatrix}$$

Comment

Step 7 of 43

Continuation of the above

$> r_3 := r_2 - t_3 \cdot u_3:$

$> r3 := evalf(Norm(r_3, infinity))$

$r3 := 0.$

$> w_4 := C.r_3:$

$> \alpha_4 := DotProduct(w_4, w_4):$

$> s_3 := \dfrac{\alpha_4}{\alpha_3}:$

$> v_4 := B.w_4 + s_3 \cdot v_3:$

$> v4 := evalf(v_4)$

$$v4 := \begin{bmatrix} 0. \\ 0. \\ 0. \end{bmatrix}$$

Result: Thus, the required solution is given by

$$\boxed{\begin{array}{l} \mathbf{x}^{(3)} = (0.06185567010, -0.1958762887, 0.6185567010)^t, \\ \left\|\mathbf{r}^{(3)}\right\|_\infty = 0 \end{array}}$$

Comment

Step 8 of 43

b. Consider the linear system of equations $\left\{\begin{array}{c} 10x_1 - x_2 + 0x_3 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ 0x_1 - 2x_2 + 10x_3 = 6 \end{array}\right\}$

Use Maple to solve the linear system with $TOL = 10^{-3}$ in the l_∞ norm.

Step1: compute the first iteration when $k = 1$

$> Tolerance := 10^{-3}:$

$> with(LinearAlgebra):$

$> A := Matrix([[10,-1,0],[-1,10,-2],[0,-2,10]]):$

$> b := \langle 9,7,6 \rangle:$

$> C := Matrix([[1,0,0],[0,1,0],[0,0,1]]):$

$> B := Transpose(C):$

Comment

Step 9 of 43

Continuation of the above

$> x_0 := \langle 0,0,0 \rangle:$

$> r_0 := b - A.x_0:$

$> w_1 := C.r_0:$

$> v_1 := B.w_1:$

$> \alpha_1 := DotProduct(w_1, w_1):$

$> u_1 := A.v_1:$

$> t_1 := \dfrac{\alpha_1}{DotProduct(v_1, u_1)}:$

$> x_1 := x_0 + t_1 \cdot v_1:$

$> x1 := evalf(x_1)$

$$x1 := \begin{bmatrix} 1.093704246 \\ 0.8506588580 \\ 0.7291361640 \end{bmatrix}$$

Continuation of the above

$> r_1 := r_0 - t_1 \cdot u_1:$

$> r1 := evalf(Norm(r_1, infinity))$

$r1 := 1.086383602$

$> w_2 := C.r_1:$

$> \alpha_2 := DotProduct(w_2, w_2):$

$> s_1 := \dfrac{\alpha_2}{\alpha_1}:$

$> v_2 := B.w_2 + s_1 \cdot v_1:$

$> v2 := evalf(v_2)$

$$v2 := \begin{bmatrix} -0.9540332141 \\ 1.148327185 \\ 0.4981896679 \end{bmatrix}$$

Comment

Step 10 of 43

Step2: compute the second iteration when $k = 2$

$> u_2 := A.v_2:$

$> t_2 := \dfrac{\alpha_2}{DotProduct(v_2, u_2)}:$

$> x_2 := x_1 + t_2 \cdot v_2:$

$> x2 := evalf(x_2)$

$$x2 := \begin{bmatrix} 0.9993129509 \\ 0.9642734456 \\ 0.7784266575 \end{bmatrix}$$

Comment

Step 11 of 43

Continuation of the above

$> r_2 := r_1 - t_2 \cdot u_2:$

$> r2 := evalf(Norm(r_2, infinity))$

$r2 := 0.1442803160$

$> w_3 := C.r_2:$

$> \alpha_3 := DotProduct(w_3, w_3):$

$> s_2 := \dfrac{\alpha_3}{\alpha_2}:$

$> v_3 := B.w_3 + s_2 \cdot v_2:$

$> v3 := evalf(v_3)$

$$v3 := \begin{bmatrix} -0.04024583189 \\ -0.07285883359 \\ 0.1502279759 \end{bmatrix}$$

Comment

Step 12 of 43

Step3: compute the third iteration when $k = 3$

$> u_3 := A.v_3:$

$> t_3 := \dfrac{\alpha_3}{DotProduct(v_3, u_3)}:$

$> x_3 := x_2 + t_3 \cdot v_3:$

$> x3 := evalf(x_3)$

$$x3 := \begin{bmatrix} 0.9957894737 \\ 0.9578947368 \\ 0.7915789474 \end{bmatrix}$$

Comment

Step 13 of 43

Continuation of the above

$> r_3 := r_2 - t_3 \cdot u_3:$

$> r3 := evalf(Norm(r_3, infinity))$

$r3 := 0.$

$> w_4 := C.r_3:$

$> \alpha_4 := DotProduct(w_4, w_4):$

$> s_3 := \dfrac{\alpha_4}{\alpha_3}:$

$> v_4 := B.w_4 + s_3 \cdot v_3:$

$> v4 := evalf(v_4)$

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Result: Thus, the required solution is given by

$$\boxed{\begin{array}{l} x^{(3)} = (0.9957894737, 0.9578947368, 0.7915789474)^t, \\ \|r^{(3)}\|_\infty = 0 \end{array}}$$

c. Consider the linear system of equations $\begin{Bmatrix} 10x_1 + 5x_2 + 0x_3 + 0x_4 = 6, \\ 5x_1 + 10x_2 - 4x_3 + 0x_4 = 25, \\ 0x_1 - 4x_2 + 8x_3 - x_4 = -11, \\ 0x_1 + 0x_2 - x_3 + 5x_4 = -11 \end{Bmatrix}$

Use Maple to solve the linear system with $TOL = 10^{-3}$ in the l_∞ norm.

Step1: compute the first iteration when $k = 1$

> $Tolerance := 10^{-3}$:

> $with(LinearAlgebra)$:

> $A := Matrix([[10, 5, 0, 0], [5, 10, -4, 0], [0, -4, 8, -1], [0, 0, -1, 5]])$:

> $b := \langle 6, 25, -11, -11 \rangle$:

> $C := Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])$:

> $B := Transpose(C)$:

Comment

Step 14 of 43

Continuation of the above

> $x_0 := \langle 0, 0, 0, 0 \rangle$:

> $r_0 := b - A.x_0$:

> $w_1 := C.r_0$:

> $v_1 := B.w_1$:

> $\alpha_1 := DotProduct(w_1, w_1)$:

> $u_1 := A.v_1$:

> $t_1 := \dfrac{\alpha_1}{DotProduct(v_1, u_1)}$:

> $x_1 := x_0 + t_1 \cdot v_1$:

> $x1 := evalf(x_1)$

$$x1 := \begin{bmatrix} 0.4654239327 \\ 1.939266386 \\ -0.8532772099 \\ -0.8532772099 \end{bmatrix}$$

Comment

Step 15 of 43

Continuation of the above

> $r_1 := r_0 - t_1 \cdot u_1$:

> $r1 := evalf(Norm(r_1, infinity))$

$r1 := 8.350571257$

> $w_2 := C.r_1$:

> $\alpha_2 := DotProduct(w_2, w_2)$:

> $s_1 := \dfrac{\alpha_2}{\alpha_1}$:

> $v_2 := B.w_2 + s_1 \cdot v_1$:

> $v2 := evalf(v_2)$

$$v2 := \begin{bmatrix} -7.455132380 \\ 3.598102958 \\ 1.088368072 \\ -9.228529102 \end{bmatrix}$$

Step2: compute the second iteration when $k = 2$

> $u_2 := A.v_2$:

> $t_2 := \dfrac{\alpha_2}{DotProduct(v_2, u_2)}$:

> $x_2 := x_1 + t_2 \cdot v_2$:

> $x2 := evalf(x_2)$

$$x2 := \begin{bmatrix} -0.7290954111 \\ 2.515782451 \\ -0.6788904051 \\ -2.331943981 \end{bmatrix}$$

Comment

Step 16 of 43

Continuation of the above

> $r_2 := r_1 - t_2 \cdot u_2$:

> $r2 := evalf(Norm(r_2, infinity))$

$r2 := 2.162309064$

> $w_3 := C.r_2$:

> $\alpha_3 := DotProduct(w_3, w_3)$:

> $s_2 := \dfrac{\alpha_3}{\alpha_2}$:

> $v_3 := B.w_3 + s_2 \cdot v_2$:

> $v3 := evalf(v_3)$

$$v3 := \begin{bmatrix} 0.3923428247 \\ 0.9263886567 \\ 2.208981638 \\ -0.4149182405 \end{bmatrix}$$

Step3: compute the third iteration when $k = 3$

> $u_3 := A.v_3$:

> $t_3 := \dfrac{\alpha_3}{DotProduct(v_3, u_3)}$:

> $x_3 := x_2 + t_3 \cdot v_3$:

> $x3 := evalf(x_3)$

$$x3 := \begin{bmatrix} -0.6711292785 \\ 2.652650425 \\ -0.3525275552 \\ -2.393245486 \end{bmatrix}$$

Comment

Step 17 of 43

Continuation of the above

> $r_3 := r_2 - t_3 \cdot u_3$:

> $r3 := evalf(Norm(r_3, infinity))$

$r3 := 0.6136998773$

> $w_4 := C.r_3$:

> $\alpha_4 := DotProduct(w_4, w_4)$:

> $s_3 := \dfrac{\alpha_4}{\alpha_3}$:

> $v_4 := B.w_4 + s_3 \cdot v_3$:

> $v4 := evalf(v_4)$

$$v4 := \begin{bmatrix} -0.4936900765 \\ 0.5566156405 \\ 0.3656462164 \\ 0.5520777968 \end{bmatrix}$$

Comment

Step 18 of 43

Step4: compute the fourth iteration when $k = 4$

> $u_4 := A.v_4$:

> $t_4 := \dfrac{\alpha_4}{DotProduct(v_4, u_4)}$:

> $x_4 := x_3 + t_4 \cdot v_4$:

> $x4 := evalf(x_4)$

$$x4 := \begin{bmatrix} -0.7976470588 \\ 2.795294118 \\ -0.2588235294 \\ -2.251764706 \end{bmatrix}$$

Comment

Step 19 of 43

Continuation of the above

> $r_4 := r_3 - t_4 \cdot u_4$:

> $r4 := evalf(Norm(r_4, infinity))$

$r4 := 0.$

> $w_5 := C.r_4$:

> $\alpha_5 := DotProduct(w_5, w_5)$:

> $s_4 := \dfrac{\alpha_5}{\alpha_4}$:

> $v_5 := B.w_5 + s_4 \cdot v_4$:

> $v5 := evalf(v_5)$

$$v5 := \begin{bmatrix} 0. \\ 0. \\ 0. \\ 0. \end{bmatrix}$$

Result: Thus, the required solution is given by

$$\boxed{\begin{array}{l} x^{(4)} = (-0.7976470588, 2.795294118, -0.2588235294, -2.251764706)^t, \\ \|r^{(4)}\|_\infty = 0 \end{array}}$$

Comment

Step 20 of 43

d. Consider the linear system of equations $\begin{Bmatrix} 4x_1 + x_2 - x_3 + x_4 = -2, \\ x_1 + 4x_2 - x_3 - x_4 = -1, \\ -x_1 - x_2 + 5x_3 + x_4 = 0, \\ x_1 - x_2 + x_3 + 3x_4 = 1 \end{Bmatrix}$

Use Maple to solve the linear system with $TOL = 10^{-3}$ in the l_∞ norm.

Step1: compute the first iteration when $k = 1$

> $Tolerance := 10^{-3}$:

> $with(LinearAlgebra)$:

> $A := Matrix([[4, 1, -1, 1], [1, 4, -1, -1], [-1, -1, 5, 1], [1, -1, 1, 3]])$:

> $b := \langle -2, -1, 0, 1 \rangle$:

> $C := Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])$:

> $B := Transpose(C)$:

> $x_0 := \langle 0, 0, 0, 0 \rangle$:

> $r_0 := b - A.x_0$:

> $w_1 := C.r_0$:

Continuation of the above

> $v_1 := B.w_1$:

> $\alpha_1 := DotProduct(w_1, w_1)$:

> $u_1 := A.v_1$:

> $t_1 := \dfrac{\alpha_1}{DotProduct(v_1, u_1)}$:

> $x_1 := x_0 + t_1 \cdot v_1$:

> $x1 := evalf(x_1)$

$$x1 := \begin{bmatrix} -0.4800000000 \\ -0.2400000000 \\ 0. \\ 0.2400000000 \end{bmatrix}$$

Comment

Step 21 of 43

Continuation of the above

> $r_1 := r_0 - t_1 \cdot u_1$:

> $r1 := evalf(Norm(r_1, infinity))$

$r1 := 0.9600000000$

> $w_2 := C.r_1$:

> $\alpha_2 := DotProduct(w_2, w_2)$:

> $s_1 := \dfrac{\alpha_2}{\alpha_1}$:

> $v_2 := B.w_2 + s_1 \cdot v_1$:

> $v2 := evalf(v_2)$

$$v2 := \begin{bmatrix} -0.6336000000 \\ 0.4032000000 \\ -0.9600000000 \\ 0.7968000000 \end{bmatrix}$$

Comment

Step 22 of 43

Step2: compute the second iteration when $k = 2$

> $u_2 := A.v_2$:

> $t_2 := \dfrac{\alpha_2}{DotProduct(v_2, u_2)}$:

> $x_2 := x_1 + t_2 \cdot v_2$:

> $x2 := evalf(x_2)$

$$x2 := \begin{bmatrix} -0.7071108901 \\ -0.09547488812 \\ -0.3441074092 \\ 0.5256091497 \end{bmatrix}$$

Comment

Step 23 of 43

Continuation of the above

> $r_2 := r_1 - t_2 \cdot u_2$:

> $r2 := evalf(Norm(r_2, infinity))$

$r2 := 0.3923421183$

> $w_3 := C.r_2$:

> $\alpha_3 := DotProduct(w_3, w_3)$:

> $s_2 := \dfrac{\alpha_3}{\alpha_2}$:

> $v_3 := B.w_3 + s_2 \cdot v_2$:

> $v3 := evalf(v_3)$

$$v3 := \begin{bmatrix} -0.08833687222 \\ 0.3612186678 \\ 0.1763742974 \\ 0.5581692536 \end{bmatrix}$$

Comment

Step 24 of 43

Step3: compute the third iteration when $k = 3$

> $u_3 := A.v_3$:

> $t_3 := \dfrac{\alpha_3}{DotProduct(v_3, u_3)}$:

> $x_3 := x_2 + t_3 \cdot v_3$:

> $x3 := evalf(x_3)$

$$x3 := \begin{bmatrix} -0.7351216968 \\ 0.01906421680 \\ -0.2881807478 \\ 0.7025994672 \end{bmatrix}$$

Comment

Step 25 of 43

Continuation of the above

> $r_3 := r_2 - t_3 \cdot u_3$:

> $r3 := evalf(Norm(r_3, infinity))$

$r3 := 0.07328354899$

> $w_4 := C.r_3$:

> $\alpha_4 := DotProduct(w_4, w_4)$:

Continuation of the above

> $s_3 := \dfrac{\alpha_4}{\alpha_3}$:

> $v_4 := B.w_4 + s_3 \cdot v_3$:

> $v4 := evalf(v_4)$

$$v4 := \begin{bmatrix} -0.07289400881 \\ 0.08774411050 \\ 0.02930753243 \\ -0.04308671334 \end{bmatrix}$$

Comment

Step 26 of 43

Step4: compute the fourth iteration when $k = 4$

> $u_4 := A.v_4$:

> $t_4 := \dfrac{\alpha_4}{DotProduct(v_4, u_4)}$:

> $x_4 := x_3 + t_4 \cdot v_4$:

> $x4 := evalf(x_4)$

$$x4 := \begin{bmatrix} -0.7534246575 \\ 0.04109589041 \\ -0.2808219178 \\ 0.6917808219 \end{bmatrix}$$

Comment

Step 27 of 43

Continuation of the above

> $r_4 := r_3 - t_4 \cdot u_4$:

> $r4 := evalf(Norm(r_4, infinity))$

$r4 := 0.$

> $w_5 := C.r_4$:

> $\alpha_5 := DotProduct(w_5, w_5)$:

> $s_4 := \dfrac{\alpha_5}{\alpha_4}$:

> $v_5 := B.w_5 + s_4 \cdot v_4$:

> $v5 := evalf(v_5)$

$$v5 := \begin{bmatrix} 0. \\ 0. \\ 0. \\ 0. \end{bmatrix}$$

Result: Thus, the required solution is given by

$$\boxed{\begin{array}{l} x^{(4)} = (-0.7534246575, 0.04109589041, -0.2808219178, 0.6917808219)^t, \\ \|r^{(4)}\|_\infty = 0 \end{array}}$$

Comment

Step 28 of 43

e. Consider the linear system of equations $\begin{Bmatrix} 4x_1 + x_2 + x_3 + 0x_4 + x_5 = 6, \\ x_1 + 3x_2 + x_3 + x_4 + 0x_5 = 6, \\ x_1 + x_2 + 5x_3 - x_4 - x_5 = 6, \\ 0x_1 + x_2 - x_3 + 4x_4 + 0x_5 = 6, \\ x_1 + 0x_2 - x_3 + 0x_4 + 4x_5 = 6 \end{Bmatrix}$

Use Maple to solve the linear system with $TOL = 10^{-3}$ in the l_∞ norm

Step1: compute the first iteration when $k = 1$

> $Tolerance := 10^{-3}$:

> $with(LinearAlgebra)$:

> $A := Matrix([[4, 1, 1, 0, 1], [1, 3, 1, 1, 0], [1, 1, 5, -1, -1], [0, 1, -1, 4, 0], [1, 0, -1, 0, 4]])$:

> $b := \langle 6, 6, 6, 6, 6 \rangle$:

> $C := Matrix([[1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [0, 0, 1, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 1]])$:

> $B := Transpose(C)$:

Comment

Step 29 of 43

Continuation of the above

> $x_0 := \langle 0, 0, 0, 0, 0 \rangle$:

> $r_0 := b - A.x_0$:

> $w_1 := C.r_0$:

> $v_1 := B.w_1$:

> $\alpha_1 := DotProduct(w_1, w_1)$:

> $u_1 := A.v_1$:

> $t_1 := \dfrac{\alpha_1}{DotProduct(v_1, u_1)}$:

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$$xl := \begin{bmatrix} 1.153846154 \\ 1.153846154 \\ 1.153846154 \\ 1.153846154 \\ 1.153846154 \end{bmatrix}$$

Continuation of the above

> $r_1 := r_0 - t_1 \cdot u_1$:

> $rl := evalf(Norm(r_1, infinity))$

$rl := 2.076923077$

> $w_2 := C.r_1$:

> $\alpha_2 := DotProduct(w_2, w_2)$:

> $s_1 := \frac{\alpha_2}{\alpha_1}$:

> $v_2 := B.w_2 + s_1 \cdot v_1$:

> $v2 := evalf(v_2)$

$$v2 := \begin{bmatrix} -1.775147929 \\ -0.6213017751 \\ 0.5325443787 \\ 1.686390533 \\ 1.686390533 \end{bmatrix}$$

Step2: compute the second iteration when $k = 2$

> $u_2 := A.v_2$:

> $t_2 := \frac{\alpha_2}{DotProduct(v_2, u_2)}$:

> $x_2 := x_1 + t_2 \cdot v_2$:

> $x2 := evalf(x_2)$

$$x2 := \begin{bmatrix} 0.5335968379 \\ 0.9367588933 \\ 1.339920949 \\ 1.743083004 \\ 1.743083004 \end{bmatrix}$$

Comment

Step 30 of 43

Continuation of the above

> $r_2 := r_1 - t_2 \cdot u_2$:

> $r2 := evalf(Norm(r_2, infinity))$

$r2 := 1.316205534$

> $w_3 := C.r_2$:

> $\alpha_3 := DotProduct(w_3, w_3)$:

> $s_2 := \frac{\alpha_3}{\alpha_2}$:

> $v_3 := B.w_3 + s_2 \cdot v_2$:

> $v3 := evalf(v_3)$

$$v3 := \begin{bmatrix} -0.6031495571 \\ -0.5840272462 \\ 1.450905341 \\ -0.1426205690 \\ 0.2605414864 \end{bmatrix}$$

Comment

Step 31 of 43

Step3: compute the third iteration when $k = 3$

> $u_3 := A.v_3$:

> $t_3 := \frac{\alpha_3}{DotProduct(v_3, u_3)}$:

> $x_3 := x_2 + t_3 \cdot v_3$:

> $x3 := evalf(x_3)$

$$x3 := \begin{bmatrix} 0.3972085646 \\ 0.8046946868 \\ 1.668009516 \\ 1.710832672 \\ 1.801998414 \end{bmatrix}$$

Comment

Step 32 of 43

Continuation of the above

> $r_3 := r_2 - t_3 \cdot u_3$:

> $r3 := evalf(Norm(r_3, infinity))$

$r3 := 0.1901348136$

> $w_4 := C.r_3$:

> $\alpha_4 := DotProduct(w_4, w_4)$:

> $s_3 := \frac{\alpha_4}{\alpha_3}$:

> $v_4 := B.w_4 + s_3 \cdot v_3$:

> $v4 := evalf(v_4)$

$$v4 := \begin{bmatrix} 0.1206684503 \\ -0.2054287320 \\ 0.008875104222 \\ 0.01624933549 \\ 0.06963009431 \end{bmatrix}$$

Step4: compute the fourth iteration when $k = 4$

> $u_4 := A.v_4$:

> $t_4 := \frac{\alpha_4}{DotProduct(v_4, u_4)}$:

> $x_4 := x_3 + t_4 \cdot v_4$:

> $x4 := evalf(x_4)$

$$x4 := \begin{bmatrix} 0.4415433373 \\ 0.7292181549 \\ 1.671270317 \\ 1.716802838 \\ 1.827581194 \end{bmatrix}$$

Comment

Step 33 of 43

Continuation of the above

> $r_4 := r_3 - t_4 \cdot u_4$:

> $r4 := evalf(Norm(r_4, infinity))$

$r4 := 0.08059779606$

> $w_5 := C.r_4$:

> $\alpha_5 := DotProduct(w_5, w_5)$:

> $s_4 := \frac{\alpha_5}{\alpha_4}$:

> $v_5 := B.w_5 + s_4 \cdot v_4$:

> $v5 := evalf(v_5)$

$$v5 := \begin{bmatrix} 0.03136722630 \\ -0.06087041662 \\ 0.01915457672 \\ 0.07828951155 \\ -0.06581975356 \end{bmatrix}$$

Comment

Step 34 of 43

Step5: compute the fifth iteration when $k = 5$

> $u_5 := A.v_5$:

> $t_5 := \frac{\alpha_5}{DotProduct(v_5, u_5)}$:

> $x_5 := x_4 + t_5 \cdot v_5$:

> $x5 := evalf(x_5)$

$$x5 := \begin{bmatrix} 0.4516129032 \\ 0.7096774194 \\ 1.677419355 \\ 1.741935484 \\ 1.806451613 \end{bmatrix}$$

Comment

Step 35 of 43

Continuation of the above

> $r_5 := r_4 - t_5 \cdot u_5$:

> $r5 := evalf(Norm(r_5, infinity))$

$r5 := 0.$

> $w_6 := C.r_5$:

> $\alpha_6 := DotProduct(w_6, w_6)$:

> $s_5 := \frac{\alpha_6}{\alpha_5}$:

> $v_6 := B.w_6 + s_5 \cdot v_5$:

> $v6 := evalf(v_6)$

$$v6 := \begin{bmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{bmatrix}$$

Result: Thus, the required solution is given by

$$\boxed{\begin{array}{l} x^{(5)} = (0.4516129032, 0.7096774194, 1.677419355, 1.741935484, 1.806451613)^t, \\ \left\| r^{(5)} \right\|_\infty = 0 \end{array}}$$

Comment

Step 36 of 43

f. Consider the linear system of equations $\begin{cases} 4x_1 - x_2 + 0x_3 - x_4 + 0x_5 + 0x_6 = 0, \\ -x_1 + 4x_2 - x_3 + 0x_4 - x_5 + 0x_6 = 5, \\ 0x_1 - x_2 + 4x_3 + 0x_4 + 0x_5 - x_6 = 0, \\ -x_1 + 0x_2 + 0x_3 + 4x_4 - x_5 + 0x_6 = 6, \\ 0x_1 - x_2 + 0x_3 - x_4 + 4x_5 - x_6 = -2, \\ 0x_1 + 0x_2 - x_3 + 0x_4 - x_5 + 4x_6 = 6 \end{cases}$

Use Maple to solve the linear system by finding two iterations as shown below.

Step1: compute the first iteration when $k = 1$

> $Tolerance := 10^{-3}$:

> $with(LinearAlgebra)$:

> $A := Matrix([[4,-1,0,-1,0,0],[-1,4,-1,0,-1,0],[0,-1,4,0,0,-1],[-1,0,0,4,-1,0],[0,-1,0,-1,4,-1],[0,0,-1,0,-1,4]])$:

> $b := \langle 0,5,0,6,-2,6 \rangle$:

> $C := Matrix([[1,0,0,0,0,0],[0,1,0,0,0,0],[0,0,1,0,0,0],[0,0,0,1,0,0],[0,0,0,0,1,0],[0,0,0,0,0,1]])$:

> $B := Transpose(C)$:

Comment

Step 37 of 43

Continuation of the above

> $x_0 := \langle 0,0,0,0,0,0 \rangle$:

> $r_0 := b - A.x_0$:

> $w_1 := C.r_0$:

> $v_1 := B.w_1$:

> $\alpha_1 := DotProduct(w_1, w_1)$:

> $u_1 := A.v_1$:

> $t_1 := \frac{\alpha_1}{DotProduct(v_1, u_1)}$:

> $x_1 := x_0 + t_1 \cdot v_1$:

> $xl := evalf(x_1)$

$$xl := \begin{bmatrix} 0. \\ 1.069915254 \\ 0. \\ 1.283898305 \\ -0.4279661017 \\ 1.283898305 \end{bmatrix}$$

Comment

Step 38 of 43

Continuation of the above

> $r_1 := r_0 - t_1 \cdot u_1$:

> $rl := evalf(Norm(r_1, infinity))$

$rl := 3.349576271$

> $w_2 := C.r_1$:

> $\alpha_2 := DotProduct(w_2, w_2)$:

> $s_1 := \frac{\alpha_2}{\alpha_1}$:

> $v_2 := B.w_2 + s_1 \cdot v_1$:

> $v2 := evalf(v_2)$

$$v2 := \begin{bmatrix} 2.353813559 \\ 1.419451128 \\ 2.353813559 \\ 1.788934573 \\ 2.898744973 \\ 1.788934573 \end{bmatrix}$$

Comment

Step 39 of 43

Step2: compute the second iteration when $k = 2$

> $u_2 := A.v_2$:

> $t_2 := \frac{\alpha_2}{DotProduct(v_2, u_2)}$:

> $x_2 := x_1 + t_2 \cdot v_2$:

> $x2 := evalf(x_2)$

$$x2 := \begin{bmatrix} 1.022375671 \\ 1.686451892 \\ 1.022375671 \\ 2.060919568 \\ 0.8310997762 \\ 2.060919568 \end{bmatrix}$$

Comment

Step 40 of 43

Continuation of the above

> $r_2 := r_1 - t_2 \cdot u_2$:

> $r2 := evalf(Norm(r_2, infinity))$

$r2 := 1.130043548$

> $w_3 := C.r_2$:

> $\alpha_3 := DotProduct(w_3, w_3)$:

> $s_2 := \frac{\alpha_3}{\alpha_2}$:

> $v_3 := B.w_3 + s_2 \cdot v_2$:

> $v3 := evalf(v_3)$

$$v3 := \begin{bmatrix} -0.1302109168 \\ 1.257840636 \\ -0.1302109168 \\ -0.2291401270 \\ 0.7448739020 \\ -0.2291401270 \end{bmatrix}$$

Step3: compute the third iteration when $k = 3$

> $u_3 := A.v_3$:

> $t_3 := \frac{\alpha_3}{DotProduct(v_3, u_3)}$:

> $x_3 := x_2 + t_3 \cdot v_3$:

> $x3 := evalf(x_3)$

$$x3 := \begin{bmatrix} 0.9907832820 \\ 1.991635138 \\ 0.9907832820 \\ 2.005324506 \\ 1.011824606 \\ 2.005324506 \end{bmatrix}$$

Comment

Step 41 of 43

Continuation of the above

> $r_3 := r_2 - t_3 \cdot u_3$:

> $r3 := evalf(Norm(r_3, infinity))$

$r3 := 0.04501427390$

> $w_4 := C.r_3$:

> $\alpha_4 := DotProduct(w_4, w_4)$:

> $s_3 := \frac{\alpha_4}{\alpha_3}$:

> $v_4 := B.w_4 + s_3 \cdot v_3$:

> $v4 := evalf(v_4)$

$$v4 := \begin{bmatrix} 0.03346224392 \\ 0.03036949440 \\ 0.03346224392 \\ -0.01933116859 \\ -0.04293045025 \\ -0.01933116859 \end{bmatrix}$$

Comment

Step 42 of 43

Step4: compute the fourth iteration when $k = 4$

> $u_4 := A.v_4$:

> $t_4 := \frac{\alpha_4}{DotProduct(v_4, u_4)}$:

> $x_4 := x_3 + t_4 \cdot v_4$:

> $x4 := evalf(x_4)$

$$x4 := \begin{bmatrix} 1. \\ 2. \\ 1. \\ 2. \\ 1. \\ 2. \end{bmatrix}$$

Comment

Step 43 of 43

Continuation of the above

> $r_4 := r_3 - t_4 \cdot u_4$:

> $r4 := evalf(Norm(r_4, infinity))$

$r4 := 0.$

> $w_5 := C.r_4$:

> $\alpha_5 := DotProduct(w_5, w_5)$:

> $s_4 := \frac{\alpha_5}{\alpha_4}$:

> $v_5 := B.w_5 + s_4 \cdot v_4$:

> $v5 := evalf(v_5)$

$$v5 := \begin{bmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{bmatrix}$$

Result: Thus, the required solution is given by

$$\boxed{\begin{array}{l} x^{(4)} = (1.000000000, 2.000000000, 1.000000000, 2.000000000, 1.000000000, 2.000000000)^t, \\ \left\| r^{(4)} \right\|_\infty = 0 \end{array}}$$

Comment

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